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Letters

Slot Coupling Between Uniform Rectangular Waveguides

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Abstract—**The results of a recent paper, which analyzes the slot-coupled waveguide problem using a 'reaction' method, are shown to be at variance with those of more established theories, for the particular case of a centrally located transverse slot in the common broad wall separating a pair of rectangular waveguides.**

The boundary value problem comprising a pair of contiguous uniform rectangular waveguides connected electromagnetically by an aperture in the common wall, is a classical problem which has received considerable attention in the literature [1]–[7]. In general, the increasingly elaborate methods of solution which are presented have enabled more complex geometries to be examined, and more accurate results to be achieved. The more recent techniques, which rely on variation [5], [6] and moment [7] methods, have evolved, and become popular, as a direct consequence of the increasing availability of high-speed digital computers.

Calculations have been performed, using several of the above methods, of the coupling coefficient c_{10} (see Fig. 1), associated with a centrally located transverse slot in the common broad wall between a pair of identical rectangular waveguides. The results of these calculations are presented in Fig. 2.

For nonresonant slots ($l<0.4\lambda$), Bethe's small aperture theory [1], modified by the resonance correction suggested by Levy [8], is a well-established and reliable analytical tool. The variational method of Sangster [5], and the moment method of Vu Khac [7], are in good agreement with the Bethe predictions over this range of slot sizes. The measured polarizabilities of Cohn [9] have been employed in the Bethe calculations to achieve this measure of agreement.

The curve of c_{10} versus slot length generated using the quasi-static antenna method, due to Lewin [3], is in general agreement with the variational and moment-method results, except

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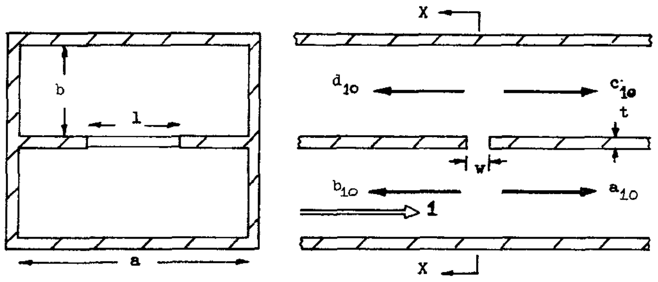

Fig. 1. Waveguide coupler schematic. (a) End section on XX. (b) Side section.

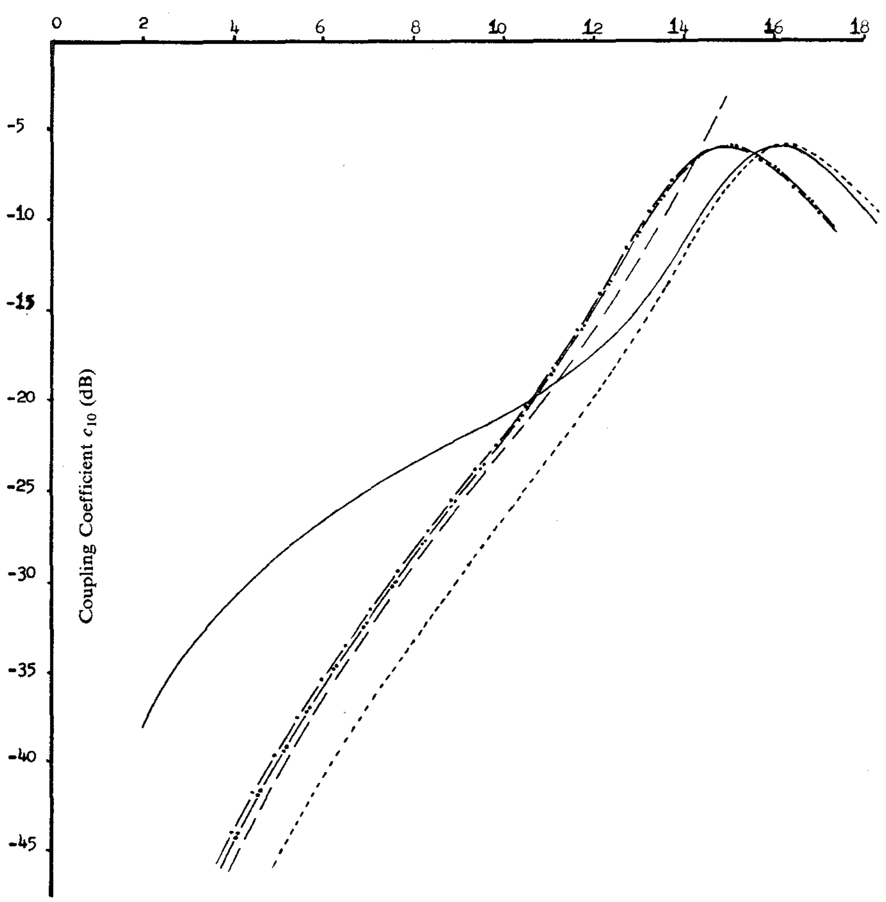

Fig. 2. Coupling versus slot length for a centrally located transverse slot in the common broad wall between a pair of rectangular waveguides, with $a=22.86$ mm, $b=10.16$ mm, $w=1.5875$ mm, $t=0.0$ mm, $\lambda=32.0$ mm.
— — Bethe theory with resonance correction
—·— Variational method [5]
—··— Moment method [7]
------- Lewin [3]
—— Reaction method [10]

for an overall shift of the curve towards longer slot lengths by a distance of approximately $\lambda/30$. This discrepancy can be attributed to shortcomings in the strip antenna model, which are discussed by Lewin. This technique predicts that resonance will occur at $l=\lambda/2$, and that at resonance $c_{10}=0.5$. The variational and moment methods agree with this latter figure but predict that resonance will occur at $l=0.47\lambda$, a result which is supported by Oliner [4], when resonance is interpreted as the condition for zero susceptance at the slot.

Superimposed on Fig. 2 is a curve of c_{10} versus slot length l, which has been produced using the "reaction" method of Pandharipande and Das [10]. While showing the correct general behavior near resonance, these new results are considerably at variance with those produced by the more established methods for slots which are small compared to the free-space wavelength. The reaction method was originally propounded by Das and Sanyal [11] as a means of analyzing long slots ($l > \lambda/2$). The suggested trial function for the E-field in the slot, while obviously appropriate for slots in this category, appears to be seriously in error for short slots. This is supported by Vu Khac's computations, which show [7] that for short nonresonant slots the aperture field is almost perfectly cosinusoidal. In addition, the effect of the side wall on the energy stored inside the waveguide is ignored in this "reaction" method, and it is suggested that for small slots which are almost purely reactive this could be a significant omission.

It is perhaps pertinent to point out here that the specific suggestion, made in [10], that the variational method of [5] is significantly in error near resonance, is patently incorrect. The apparent discrepancy detected by Pandharipande can be traced to an unfortunate printing error in the published paper. This error can be isolated without difficulty by performing a dimensional check on eq. (34), or by consulting fig. 4 of [5] which was extracted directly from the original thesis describing the variational method [12]. Equation (34) of [5] should read

$$c_{10} = -d_{10} = \frac{AP(I_1 + h_1 J_1)^2}{(X + jY)(I_1 + h_1 J_1)^2 - j \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} Q_{mn}(I_m + h_1 J_m)^2} \tag{1}$$

where $A = 1$ is the amplitude of the incident wave,

$$P = \frac{F^2 ab \sin^2 \beta w \sin^2 \frac{\pi s}{a}}{\omega \epsilon_0}$$

$$X = \frac{2}{\beta^3}\left(1 - \frac{\pi^2}{a^2 k_0^2}\right) \sin^2 \frac{\pi s}{a} (\cos 2\beta w - 1)$$

$$Y = \frac{2}{\beta^3}\left(1 - \frac{\pi^2}{a^2 k_0^2}\right) \sin^2 \frac{\pi s}{a} (2\beta w - \sin 2\beta w)$$

$$Q_{mn} = \frac{c_0}{\alpha_{mn}^3}\left(1 - \frac{m^2\pi^2}{a^2 k_0^2}\right) \sin^2 \frac{m\pi s}{a} \cdot [4\alpha_{mn} w - 2(1 - \exp(-2\alpha_{mn} w))]$$

$$c_0 \begin{cases} = 0, & \text{for } n = 0; m = 1 \\ = 1, & \text{for } n = 0; m > 1 \\ = 2, & \text{for } n > 0; m > 1 \end{cases}$$

$$I_m = \int_{-\pi/2}^{\pi/2} \cos m\alpha\theta \cos \theta \, d\theta = \begin{cases} \pi/2, & \text{when } m\alpha = \frac{2ml}{a} = 1 \\ \frac{2}{1 - m^2\alpha^2} \cos \frac{m\pi\alpha}{2}, & \text{when } m\alpha \neq 1 \end{cases}$$

$$J_m = \int_{-\pi/2}^{\pi/2} \cos m\alpha\theta \cos 3\theta \, d\theta = \begin{cases} \pi/2, & \text{when } m\alpha = 3 \\ \frac{6}{m^2\alpha^2 - 9} \cos \frac{m\pi\alpha}{2}, & \text{when } m\alpha \neq 3. \end{cases}$$

At resonance the coupling coefficient c_{10} will be real, which implies that the imaginary terms in the denominator of (1) must sum to zero. Thus at resonance the variational method gives

$$c_{10} = \frac{P}{X} = \frac{\beta^2}{2\omega\epsilon_0 k_0 Z_0 \left(1 - \frac{\pi^2}{a^2 k_0^2}\right)}. \tag{2}$$

But

$$\beta^2 = k_0^2 - \frac{\pi^2}{a^2} \qquad \text{and} \qquad k_0 Z_0 = \omega \mu_0$$

thus at resonance, $c_{10} = 0.5 = -6$ dB, and this is in exact agreement with Pandharipande's resonance value for c_{10}.

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Comments on "Coupling of Waveguides Through Large Aperture"

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